This exam is due in on March 19 by 4:30 p.m. Turn in to Mohan Singh in SocSci 416. Concise answers are given extra credit. Assertions that are unproved are given zero credit even if they are correct.

Problem 1 (50%)

Consider the following assignment problem. The output of worker $i$ with firm $j$ is $a_{ij}$. There are $N$ workers and $M$ firms. The output of worker $i$ alone is $\varphi_i$. The output of firm $j$ alone is $\eta_j$. The matches are 1-1 (one worker with one firm).

1. Characterize the optimal assignment. Derive optimal wages ($w_i$) and profits ($\pi_j$). Analyze two cases

   (a) $\varphi_i = 0 \forall i$ and $\eta_j = 0 \forall j$,

   (b) $\varphi_i \neq 0 \forall i$, $\varphi_i \neq \varphi_{i'}$, $i \neq i'$ and $\eta_j \neq 0 \forall j$, $\eta_j \neq \eta_{j'}$, $j \neq j'$.

In particular, discuss the following questions: Are the optimal wages and profits unique? What is the zone of indeterminacy if they are not unique? If $N > M$, which workers are unemployed?

2. Now suppose that we change the problem so that $a_{ij} = \ell_i^a c_j^\beta$, with $\ell_i \in [0, 1]$, $c_j \in [0, 1]$, $\ell \sim F_\ell$, $c \sim F_c$, where we write $F_x$ for the cdf of a random variable $x$. Assume 1-1 matches. What is the output maximizing allocation? Derive the optimal pricing and profits equations that support it. Are wages and profits unique? For specificity assume $F_\ell = \ell^a$, $F_c = c^b$, where $a > 0$, $b > 0$ are parameters, and $N > M$. Assume that the workers of quality $\ell$ who are unmatched produce produce $g(\ell)$; unmatched firms of quality $c$ produce $\mu(c)$. Which workers (or firms) are unemployed? (Consider two cases: $g(\ell)$ increasing in $\ell$; $\mu(c)$ increasing in $c$ and $g(\ell) = \bar{g} \forall \ell$, $\mu(c) = \bar{\mu} \forall c$.) Characterize how the wage and profit functions depend on $a$, $b$, $N$, $M$, $\alpha$, $\beta$ (derive the wage function).

   (a) Is it necessary to assume $0 \leq \alpha + \beta \leq 1$? Why or why not?

   (b) If the hedonic wage function $W(\ell)$ is convex increasing in $\ell$, is the profit function $\pi(c)$ necessarily concave increasing in $c$? Be precise.
3. What restriction on $a_{ij}$ in the first part produces the technology used in the second part?

4. Suppose now that workers have preferences $U(y, e)$ where $y$ is consumption and $e$ is effort. $\ell$ in part (2) is now interpreted as an endowment,

$$U(y, e) = y - V(e, \ell)$$

Disutility of effort:

$$V(e, \ell) = e^k \ell, \quad k \geq 1.$$  

The greater $\ell$, the greater the disutility of work. Firms hire $e$ (effort). $V$ is convex increasing in $e$. All workers have the same $k$ but differ in $\ell$. Firm output $G$ is given by

$$G(e, c) = A(e)^{\alpha} (c)^{\beta}, \quad \alpha, \beta > 0; \quad A \geq 0.$$  

Firms are identified by endowments of $c$. The distributions of $c$ and $\ell$ are as before (in part (2)).

(a) Derive the hedonic wage and profit functions in terms of $\alpha, \beta, k, N, M, a, b$ and characterize the equilibrium.

(b) What parameters are identified from the hedonic wage and profit functions.

(c) Evaluate Rosen’s two-stage estimation method based on (a). Does it identify technology and distribution in this setting? Show why or why not.

5. Suppose that you observe equilibrium allocations from the discrete economy you characterized in part (1). You cannot nonparametrically identify the $w_i$ and $\pi_j$ from a single equilibrium allocation. Show why. Give conditions under which you can identify these parameters from a single market. Be as precise as you can be.
Problem 2 (50%)

Consider the Mincer Earnings Function:

\[
\ln y = \alpha_0 + \alpha_1 S + \alpha_2 X + \alpha_3 X^2 + \varepsilon.
\]

Where \( E(\varepsilon) = 0 \), \( S \) is years of schooling, \( X \) is experience (years of work since leaving school). In light of the papers you have read and discussed in this course, what is the evidence on the following issues.

1. Ignoring ability bias or endogeneity of \( S, X \), does \( \alpha_1 \), an OLS estimate of schooling, estimate a return to schooling? Define these concepts in a world of equalizing differentials, in a world of uncertainty and in a world of the sequential resolution of uncertainty. What evidence if any can you cite? Give theoretical and empirical arguments. Be concise. Do not regurgitate the “50 Years of Mincer Earnings Functions” paper. Add to it.

2. What is the evidence on the linearity of the equation in terms of \( S \)? Why is this important? Discuss Card (1999, 2001) as listed on the reading list, and other sources. What are the economic and econometric implications of this linearity? Contrast this with an ordered discrete choice model and an unordered model. Contrast the Card model with Carneiro, Heckman and Vytlacil (2003) and Cameron and Heckman (1998) in terms of exclusion restrictions. What is \( LATE \) in Card’s model? Discuss the validity of his and other instruments.

3. What is the evidence on the importance of ability bias and comparative advantage in the labor market. Be concise and precise. Compare and contrast Card (1999, 2001) with Carneiro, Heckman and Vytlacil (2003). (Specifically, does \( IV > OLS \) for schooling coefficient imply credit constraints?) Discuss the quality of the instruments.

4. Justify the role of \( X \) in the Mincer equation. What do the coefficients measure?

5. Suppose the market interest rate goes up. How would the coefficients of this equation change? What if tuition increases? Family incomes increase? Use whatever models you know to answer these questions precisely.